BGSMath

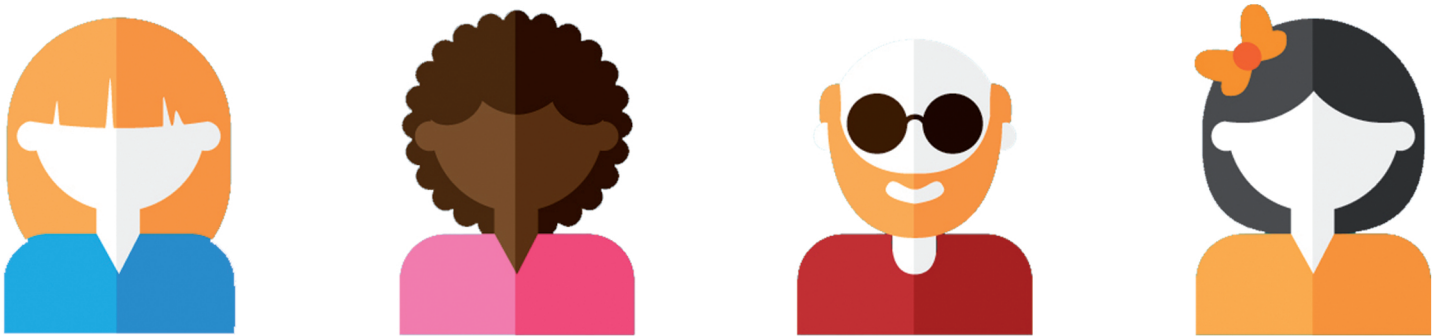
BARCELONA GRADUATE SCHOOL OF MATHEMATICS

II Junior Meeting

Friday, May 13, 2016
9 a.m. to 6 p.m.

Institut d'Estudis Catalans
(Sala Prat de la Riba)
Carrer del Carme 47, 08001, Barcelona

Organized by:
Carlos Domingo (UB)
Azahara de la Torre (UPC)
Isabel Serra (CRM, UAB).



II BGSMath Junior Meeting

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09:00 - 09:20	**Registration**
09:20 - 09:30	**Short Presentation**
09:30 - 10:00	Víctor Navas
10:00 - 10:30	Francesc de Borja Pons Llopis
10:30 - 11:00	Daniel Seco
11:00 - 11:30	**Coffee Break**
11:30 - 12:00	Núria Folguera Blasco
12:00 - 12:30	Walter Andrés Ortiz Vargas
12:30 - 13:00	Rodrigo Gonçalves Schaefer
13:00 - 14:30	**Time for lunch (not included)**
14:30 - 15:00	Ángel Arroyo García
15:00 - 15:30	Anel Nurtay
15:30 - 16:00	Gemma Colldeforns Papiol
16:00 - 16:30	**Coffee Break**
16:30 - 17:00	Roc Oliver
17:00 - 17:30	Petr Chunaev
17:30 - 18:00	Joan Gimeno

Avalanches in the zero-temperature Random Field Ising Model: influence of the aspect ratio and boundary conditions on universal finite-size scaling functions

Víctor Navas, CRM

In this talk we will introduce a paradigmatic model that reproduces avalanche dynamics in disordered systems: the zero-temperature Random Field Ising Model (RFIM). Recent results concerning the dependence of universal finite size scaling functions in a two-dimensional (2D) system with boundary conditions of different nature and different aspect ratios will be presented. A convenient scaling variable that accounts for the deformation of the distance to the critical point caused by the aspect ratio is chosen. In addition, assuming that the dependence of the finite size scaling functions on the aspect ratio can be accounted for by an additional multiplicative factor, collapses of data are obtained for different system sizes, different aspect ratios, and different types of the boundary conditions into a single scaling function.

Particle Filtering for applications in Data Assimilation

Francesc de Borja Pons Llopis, Imperial College London

Data Assimilation aims to provide estimates for spatial and temporal varying phenomena based on a sequence of observations. This task is essential in areas such as meteorology and can be mathematically formulated as high dimensional Filtering problems. Since in most cases analytical solution is unattainable, numerical methods are necessary. However, methods which have well understood error bounds suffer from the curse of dimensionality and for those commonly used, is not possible to characterize the uncertainty in the estimates. The state of the art of principled methods for high dimensional filtering problems are Particle Filters. Combined with tempering and carefully crafted MCMC steps, they have proven to be able to handle complicated high dimensional problems. They also have the additional advantage that computations can be highly parallelised. My work focuses in the problem of implementing this method for the two-dimensional Stochastic Navier Stokes equations.

Cyclicity and optimal polynomials

Daniel Seco, UB

I will start by presenting a classical problem in complex analysis of determining whether a function is cyclic in a given function space. Then I will show the approach of our team, which reduces this problem to studying a sequence of polynomials that we call optimal approximants, and if there is time, some new results about properties of these polynomials.

Robustness of epigenetic states: the roles of oncometabolic transformation and aging

Núria Folguera Blasco, CRM

In this talk, I will present a stochastic model of epigenetic regulation that will allow to analyse the robustness of epigenetic states. In particular, the model will be focused on the coupled effects of oncometabolic transformation, which yields to a reduced activity of the histone demethylases (HDMs) which, in turn, implies that the differentiation genes are locked in a silent state, and aging, which leads to a decrease on the activity of the histone deacetylases (HDACs). Both factors are known to be related to cancer although their relationship remains to be addressed. I will show that the robustness of epigenetic states is the outcome of the complex interaction between the activity of HDMs and HDACs. In order to do so, multi-scale optimal path theory and the semi-classical quasi-steady state approximation will be applied. Numerical results obtained with the Gillespie stochastic simulation algorithm will be shown too. These can help to shape novel therapeutic interventions in oncology.

On the Cauchy profile: Isoperimetry and Symmetrization

Walter Andrés Ortiz Vargas, UAB

In the theory dedicated to metric spaces of probability, obtaininig Sobolev-Poincaré type inequalities, using isoperimetry and symmetrization. It is noteworthy that these have been studied for the isoperimetric concave profile; currently we focus in this type of isoperimetric inequalities with convex profile, adapting these characterizations in probability spaces with invariant rearrangement, whose measure is the Cauchy.

Global instability in Hamiltonian Systems

Rodrigo Gonçalves Schaefer, UPC

In this work we illustrate the Arnold diffusion in a concrete example: the *a priori* unstable Hamiltonian system of $2 + 1/2$ degrees of freedom

$$H(p, q, I, \varphi, s) = \frac{p^2}{2} + \cos q - 1 + \frac{I^2}{2} + h(q, \varphi, s; \varepsilon),$$

proving that for *any* small periodic perturbation of the form

$$h(q, \varphi, s; \varepsilon) = \varepsilon \cos q \, (a_{00} + a_{10} \cos \varphi + a_{01} \cos s)$$

($a_{10}a_{01} \neq 0$ and $\varepsilon \neq 0$ small enough) there is global instability for the action, i.e., $I(0) \leqslant -I_\varepsilon < I_\varepsilon \leqslant I(T)$ for some T and for any positive $I_\varepsilon \leqslant C \log \frac{1}{\varepsilon}$ for some constant C. For this, we apply a geometrical mechanism based in the so-called Scattering map.

Mean value properties for p-harmonic functions

Ángel Arroyo García, UAB

It is well-known that harmonic functions satisfy the so-called mean value property. Moreover, a harmonic function u in $\Omega \subset \mathbb{R}^n$ can be characterized by the asymptotic mean value property, i.e.,

$$u(x) = \frac{1}{|B(x,r)|} \int_{B(x,r)} u(y) \, dy + o(r^2).$$

Similarly, for $1 < p < \infty$, weak solutions of the p-Laplace equation,

$$\operatorname{div}(|\nabla u|^{p-2} \nabla u) = 0,$$

satisfy an asymptotic nonlinear mean value property which is given by

$$u(x) = \frac{\alpha}{2}\Big(\inf_{B(x,r)} u + \sup_{B(x,r)} u \Big) + \frac{1-\alpha}{|B(x,r)|} \int_{B(x,r)} u(y) \, dy + o(r^2),$$

where $\alpha = \frac{p-2}{n+p}$. It turns out that this characterization holds directly for $n = 2$, and in a viscosity sense for $n \geq 3$.

However, for $p \neq 2$, a new class of functions appears when we consider the nonlinear mean value property for some particular families of balls. These functions are called p-harmonius and, under certain assumptions, they approach p-harmonic functions as the radii go to zero.

Math modeling of biological evolution: appearance of species, specialist vs. generalist

Anel Nurtay, CRM

We study the biological evolution and the appearance of species. In particular, we investigate a route for the appearance of new species via the development of specializations; that is, the adaptation for a better exploiting of a particular niche at the expenses of demoting the ability of exploitation other niches. Also we inquiry whether universalism gives an advantage, and hence has a chance to arise and develop, if the supply of resources in specific niches is volatile. Due to their simplicity, viruses serve as an excellent model in Evolutionary Biology. We construct a mathematical model of viral evolution, which should be able to describe the development of specialization. This model is to be based on the Nowak-May model of HIV. It may be worthy of noting that system is equivalent to a nutrient-bacteria model (or a chemostat model) and a nutrient-plankton model. Numerical results of both deterministic and stochastic versions of the model are obtained.

Computation of market risk measures with stochastic liquidity horizon by Shannon wavelet expansions

Gemma Colldeforns Papiol, CRM

Fourier-based methods are one of the numerical approaches used in financial mathematics. We present a technique based on the Fourier transform of a density function. Based on this framework, we deal with questions regarding efficient market risk measurement taking into account the market liquidity risk. We employ an approach based on wavelets, specifically, we provide methodology for computing the Value-at-Risk (VaR) and the Expected Shortfall (ES) using SWIFT (Shannon Wavelet Inverse Fourier Technique). This technique is based on a Shannon wavelet expansion of a density function assuming that its Fourier transform is known. Shannon wavelets are smooth wavelets based on the cardinal sine function. From its nature SWIFT presents several benefits such as high accuracy, robustness and fast convergence.

Vector-valued Bergman spaces

Roc Oliver, UB

In this talk I want to describe in a general way Bergman spaces and one of their generalizations that is not so common, the vector-valued Bergman spaces. This setting generalizes many other results that are just an example of this. This work is part of my PhD thesis and will be explained in a smooth way.

A new family of singular integral operators whose L^2-boundedness implies rectifiability

Petr Chunaev, UAB

Let $E \subset \mathbb{C}$ be a Borel set such that $0 < \mathcal{H}^1(E) < \infty$. David and Léger (1999) proved that the Cauchy kernel $1/z$ (and even its coordinate parts $\Re z/|z|^2$ and $\Im z/|z|^2$), $z \in \mathbb{C}\setminus\{0\}$, has the following property: the $L^2(\mathcal{H}^1\lfloor E)$-boundedness of the corresponding integral operator implies the rectifiability of E. Recently Chousionis, Mateu, Prat and Tolsa (2012) extended this result to any kernel of the form $(\Re z)^{2n-1}/|z|^{2n}$, $n \in \mathbb{N}$. In the talk, we present our result stating that the above-mentioned property is valid for operators associated with the much wider class of the kernels $(\Re z)^{2N-1}/|z|^{2N} + t \cdot (\Re z)^{2n-1}/|z|^{2n}$, $N \geq n$, where n, N are positive integer and $t \in \mathbb{R} \setminus (t_1, t_2)$ with t_1, t_2 depending only on n and N.

First steps in Automatic Differentiation

Joan Gimeno, UB

Nowadays, Automatic Differentiation (AD) has become a useful tool so as to differentiate a given expression by a computer program. This expression can be as far as one can implement using either arithmetic operations or intrinsic functions provided by the programming environment. The talk will present the main AD ideas. In particular, the two main modes; forward and reverse. Considering combinations of the two previous modes, the first and second order approaches will show an equivalence theorem of the different AD modes. Finally, quick comments of computational aspects will be also explain as well as quick comparisons between the classical numerical differentiation and AD.

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